QUIZZES AND MIDTERMS FOR MATH 2280 INTRODUCTION TO DIFFERENTIAL EQUATIONS

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Week 2

Problem. Suppose that a population $P(t)$ has birth rate $\beta = \frac{1}{1000}P$ and constant death rate δ.

i) Find the equilibrium solution;

ii) If $P(0) = 100$ and $P'(0) = 8$, find $P(t)$;

iii) Will there be a population explotion? When?

Week 3

Problem. Consider the following differential equation

$$\frac{dy}{dx} = y^2 - 9.$$

i) Find the critical points and determine whether each critical point is stable or unstable;

ii) Find $y(x)$ which satisfies the above differential equation together with the initial condition $y(0) = 4$.

Week 4

Problem. Solve the following initial value problem

$$\begin{cases} y^{(3)} + 6y^{(2)} + 9y' = 0 \\ y(0) = 0 \\ y'(0) = 0 \\ y^{(2)}(0) = 9. \end{cases}$$

Week 5 Super Quiz

Problem 1. Consider the following differential equation

$$3y^{(2)} - 6y' + 3y = 3xe^x.$$

i) Find the general solution of the associated homogeneous differential equation.

ii) Find the general solution of the above non-homogeneous differential equation.

Problem 2. Solve the following initial value problem

$$\begin{cases} xy' = y \\ y(1) = 5 \end{cases}.$$

Midterm 1

Problem 1. Find the solution of the following initial value problem

$$\begin{cases} x' = 2y \\ y' = -2x \\ x(0) = 5 \\ y(0) = 3 \end{cases}.$$

Problem 2. Find the general solution of the following differential equation

$$y^{(3)} - 8y^{(2)} + 16y' = e^{4x}.$$

Problem 3. Use the method of variation of parameters to find a particular solution of the following differential equation

$$y^{(2)} - 8y' + 16y = e^{4x}.$$

Problem 4. Find the solution of the following initial value problem

$$\begin{cases} y' = e^{x-y} \\ y(0) = 5 \end{cases}.$$

Week 9

Problem. Apply the method of undetermined coefficients to find a particular solution of the following system

$$\begin{cases} x' = y + \sin(t) \\ y' = x + 2\sin(t) \end{cases}.$$

Week 11 Super Quiz

Problem 1. Find all critical points of the following system

$$\begin{cases} x' = 4x - x^2 - 2xy \\ y' = 3y - 2xy + y^2 \end{cases}$$

and investigate the type and stability of each.

Problem 2. i) Find the general solution of the following system

$$\begin{cases} x' = x + y \\ y' = 3x - y \end{cases}.$$

ii) Find all critical points of the above system and investigate the type and stability of each.

Midterm 2

Problem 1. Use the method of undetermined coefficients to find a particular solution of the following system

$$\begin{cases} x' = x - y + te^t \\ y' = 2x + y + 2te^t \end{cases}.$$

Problem 2. Find all critical points of the following system

$$\begin{cases} x' = y^2 - 1 \\ y' = x^2 - 1 \end{cases}$$

and investigate the type and stability of each.

Problem 3. Use Laplace transforms to solve the following initial value problem

$$\begin{cases} x'' - 4x' + 13x = 13 \\ x(0) = x'(0) = 0 \end{cases}.$$

Week 15 Quiz

Problem. Let $f(t)$ be the periodic function of period 1 defined by

$$f(t) = \cos(\pi t)$$

for $0 < t \leq 1$.

a) Compute the Fourier series of $f(t)$.

b) What is the value of the Fourier series at the points $t \in \mathbb{Z}$?

Hint: Use the following equalities:

$$\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))$$

$$\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B)).$$

Final Exam

Problem 1. Consider the following differential equation

$$y^{(4)} + 4y^{(2)} = \sin(x).$$

i) Find the general solution of the associated homogeneous differential equation.

ii) Find the general solution of the above non-homogeneous differential equation.

iii) Find the solution of the above non-homogeneous differential equation satisfying the following initial conditions:

$$y(0) = y'(0) = 0, \quad y^{(2)}(0) = 4, \quad y^{(3)}(0) = -16.$$

Problem 2. Find all critical points of the following system

$$\begin{cases} x' = x - 3x^2 + 8xy \\ y' = 8y - 2y^2 - 2xy \end{cases}$$

and investigate the type and stability of each.

Problem 3. Use Laplace transforms to solve the following initial value problem

$$\begin{cases} x'' - 8x' + 20x = 0 \\ x(0) = 2 \\ x'(0) = 16 \end{cases}.$$

Problem 4. Find formal Fourier series solutions of the following endpoint value problem

$$\begin{cases} x'' + 4x = t^2 + 1 \\ x'(0) = x'(\pi) = 0 \end{cases}.$$