

QUIZZES AND EXAMS FOR MATH 1220  
CALCULUS II

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**Week 1 Quiz**

**Problem 1.** Find the following integral

$$\int \frac{2 - 3x^2}{x^3 - 2x} dx.$$

**Problem 2.** Given

$$y = \frac{(x + 1)^{\frac{2}{3}}}{(x^2 - 3)^{\frac{1}{5}}},$$

find  $y'$  by logarithmic differentiation.

**Week 2 Quiz**

**Problem 1.** Find the following integral

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx.$$

**Problem 2.** Given

$$y = (\sqrt{x})^{\sin(x)}.$$

compute  $dy/dx$ .

**Week 3 Quiz**

**Problem 1.** Find the following limit

$$\lim_{x \rightarrow \infty} \left( \frac{x - 3}{x} \right)^{2x}.$$

**Problem 2.** Solve the following differential equation

$$\begin{cases} y' + \frac{y}{1-x} = 1 - x^2 \\ y(0) = 1 \end{cases}.$$

*Hint:* the initial condition suggests that we are interested in values of  $x$  close to 0. In particular, we can assume  $x < 1$ , hence the equality  $|1 - x| = 1 - x$  holds.

### Week 4 Super Quiz

**Problem 1.** Given

$$y = x^{2x^2},$$

compute  $dy/dx$ .

**Problem 2.** Find the following limit

$$\lim_{x \rightarrow \infty} \left( \frac{x}{x-5} \right)^{3x}.$$

**Problem 3.** Find the following integral

$$\int \frac{1}{x^2 - 4x + 16} dx.$$

**Problem 4.** Find the following integral

$$\int x \sinh(1 - 2x^2) dx.$$

### Week 5 Quiz

**Problem 1.** Evaluate the following integral

$$\int x^3 \ln x^3 dx.$$

**Problem 2.** Evaluate the following integral

$$\int \cos^5(5x) dx.$$

### Midterm 1

**Problem 1.** i) (4 points) Evaluate the following integral

$$\int \frac{1}{\sqrt{9 - 4x^2}} dx.$$

ii) (4 points) Evaluate the following integral

$$\int \frac{x}{\sqrt{9-4x^2}} dx.$$

**Problem 2.** i) (6 points) Evaluate the following integral

$$\int x^2 \sinh(x) dx.$$

ii) (6 points) Evaluate the following integral

$$\int_0^{\frac{1}{3}} t \sqrt[3]{(3t+1)^2} dt.$$

**Problem 3.** i) (6 points) Evaluate the following integral

$$\int \sin(2x) \cos^5(2x) dx.$$

ii) (6 points) Evaluate the following integral

$$\int \sin^2(3x) \cos^2(3x) dx.$$

**Problem 4.** (8 points) Solve the following differential equation

$$\begin{cases} y' + \frac{\ln(x)}{x}y = x^{1-\frac{1}{2}\ln(x)} \\ y(1) = 1 \end{cases}.$$

### Week 8 Quiz

**Problem 1.** Find the following limit

$$\lim_{x \rightarrow +\infty} \left( \cos \left( \frac{1}{x} \right) \right)^x.$$

**Problem 2.** Find the following limit

$$\lim_{x \rightarrow 0} \frac{6x - 6 \sin(x) - x^3}{x^5}.$$

**Week 9 Quiz****Problem 1.** Evaluate the following integral

$$\int_0^{\infty} \frac{1+x-x^2}{e^x} dx.$$

**Problem 2.** Evaluate the following integral

$$\int_{-3}^0 \frac{dx}{\sqrt{9-x^2}}.$$

**Week 11 Super Quiz****Problem 1.** In the following, determine whether the given series converges or diverges, and if it converges, find its sum.

$$i) \quad \sum_{k=3}^{\infty} \frac{-1}{k^2 - 3k + 2}; \quad ii) \quad \sum_{k=3}^{\infty} \left(\frac{1}{8}\right)^{2-k}; \quad iii) \quad \sum_{k=3}^{\infty} \left(\frac{1}{3}\right)^{2k+2}.$$

**Problem 2.** Find the following limit

$$\lim_{x \rightarrow 0} \left( \frac{1}{e^x - 1} - \frac{1}{x} \right).$$

**Problem 3.** Evaluate the following improper integral or show that it diverges

$$\int_1^5 \frac{dx}{\sqrt[5]{x-1}}.$$

**Week 12 Quiz****Problem.** In the following, determine whether the given series converges or diverges:

$$i) \quad \sum_{n=1}^{\infty} \frac{e^n}{(2n)!}; \quad ii) \quad \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2 - 1}{e^n}.$$

**Midterm 2****Problem 1.** i) (6 points.) Evaluate the following integral

$$\int_2^{\infty} \frac{dx}{x \cdot (\ln(x) + 1)^{\frac{3}{2}}}.$$

ii) (6 points.) Determine whether the following series converges or diverges

$$\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n) + 1)^{\frac{3}{2}}}.$$

**Problem 2.** i) (6 points.) Determine whether the following series converges or diverges

$$\sum_{n=1}^{\infty} n^{2/(3n)}.$$

ii) (6 points.) Determine whether the following series converges or diverges

$$\sum_{n=2}^{\infty} (-1)^n \frac{(\ln(n))^{3/2}}{n}.$$

**Problem 3.** (10 points.) Find the convergence set for the following power series

$$\sum_{n=3}^{\infty} \frac{2^{2n-1} x^n}{(n+1)^2}.$$

**Problem 4.** i) (6 points.) Find the sum of

$$\sum_{n=1}^{\infty} n x^{n+1}.$$

ii) (**Extra** 4 points.) Find the power series representation for the following function and specify the radius of convergence

$$f(x) = \frac{x}{4 + x^2}.$$

### Final Exam

**Problem 1.** (10 points.) Solve the following differential equation

$$\begin{cases} \frac{dy}{dx} + (y+1) \sin(3x) = 0 \\ y(0) = \sqrt[3]{e} - 1 \end{cases}.$$

**Problem 2.** i) (5 points.) Compute the following limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2} + \frac{x^2}{8}}{x - \sin(x)}.$$

i) (5 points.) Compute the following limit

$$\lim_{x \rightarrow 0} (\cos(x))^{1/x^2}.$$

**Problem 3.** Compute the following integrals

$$i) \quad (4 \text{ points}) \quad \int \frac{1}{x^2 + 2x + 3} dx,$$

$$ii) \quad (2 \text{ points}) \quad \int \frac{x + 1}{x^2 + 2x + 3} dx,$$

$$iii) \quad (4 \text{ points}) \quad \int \frac{-x - 6}{x^3 + 2x^2 + 3x} dx.$$

**Problem 4.** i) (5 points.) Determine whether the following series converges or diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3 \cdot e^{n+3}}{3^n}.$$

ii) (5 points.) Find the power series representation for the following function and specify the radius of convergence

$$f(x) = \frac{x^5}{16 + x^4}.$$

**Problem 5.** (10 points.) Find the convergence set for the following power series

$$\sum_{k=3}^{\infty} (-1)^k \frac{3^{2k}}{(k-2)^2} (x-2)^{2k}.$$

**Problem 6.** Consider the curve described by the following polar equation

$$r = 1 - 2 \cos(\theta).$$

- i) (*2 points.*) Find the tangent lines at the origin.
- ii) (*4 points.*) Sketch the graph.
- iii) (*4 points.*) Compute the area of the region inside its small loop.